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Quasidynamic model for earthquake simulations

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A cellular automaton is constructed that simulates the response of a one-dimensional dynamical Burridge-Knopoff model for earthquake occurrence. The central element of the model involves an approximation to the momentum overshoot of the static equilibrium state in the dynamic case. The model gives satisfactory comparison with, and a reduction in computation by $10N$ relative to, histories of synthetic seismicity using the dynamic model, when a reasonable choice of overshoot is selected. Solutions that use the "standard" quasistatic automaton, in which the final state of stress is set equal to the dynamic friction, give less satisfactory agreement. Thus momentum overshoot is an important factor in regulating earthquake self-organization and cannot be ignored in any accurate simulation of earthquake histories.

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Efforts to model seismicity as a problem in the physics of the self-organization of a complex dynamical system have focused on a threshold dynamics that divides time in a piecewise manner into slow and fast intervals [1]. The state of stress at the end of the fast time episode is an initial condition for the determination of the time, place, and size of future fracture events, under conditions of uniform, slow loading in slow time. In quasistatic models of seismicity the final state of stress is specified by assumption to be the dynamic friction [1,2], which it is during the slip episode itself. In dynamic models [3–5] the final state of stress is the end product of the solution to a problem in dynamics; in these cases, the final stress usually overshoots the dynamic friction, a consequence of the process of healing of the fracture. Healing takes place progressively due to the finite time for the propagation of stress waves [6] across the fracture. The momentum of the part of the fracture that continues to slip generates an overshoot stress which appears at the moving boundary between the healed and slipping parts of the fracture. The dynamical models are evidently more computationally demanding.

To investigate the importance of the dynamics of healing on the simulation of long-term evolutionary seismicity, we study the overshoot of the final stress state which depends crucially on the process of cessation of slip. In quasistatic models there is no provision for accommodation to the problem of the cessation of slip; in the dynamics, cessation of slip appears naturally. In the dynamic models, the final state of stress can be significantly different from that usually assumed in the *ad hoc* quasistatic models due to the differential momentum effects described above.

While there are a number of quasistatic models that describe earthquake fractures as the propagation of a series of dislocations, only a few models simulate the formation of extended cracks, much less their dynamics. Crack and dislocation versions of iterative quasistatic models of fracture give dramatically different histories of seismicity [1]. To date, quasistative models have not been iterated successfully because of a failure to make a provision for healing [7], or because of restrictive assumptions concerning runaways and the reset after runaways [1,8]. (See [9] for a discussion of the influence of initial conditions and reset conditions after runaways on the seismic history of a model system.) We focus on the more realistic models of earthquake fractures as extended cracks with correlations of slip along the fracture surface.

In many models the assumption is made that the transfer of stress takes place through nearest-neighbor coupling as a static or dynamic cellular automation, instead of through the agency of the long range forces of elasticity; we do not try to redress this assumption in this paper. Although it is not to be expected that any simulation that involves short range stress transfer will replicate the natural paradigm, nevertheless it should be possible to assess the relative importance of introducing dynamics into these models, and that is what we propose to do in this paper. We compare quasistatic and dynamic extended crack models with nearest-neighbor coupling in both cases, by describing a quasidynamic model with a tunable parameter that controls the amount of overshoot; in the case of zero overshoot, we recover the quasistatic assumption. We demonstrate that, aside from other deficiencies that may prohibit a simulation from replicating the earthquake paradigm, any model that claims to simulate earthquake activity properly must account for the way in which the dynamics of rupture and healing can influence the final state of stress across the ruptured fault.

We offer a set of automation rules that can be used to develop self-organizing, extended-crack models of iterative earthquake phenomena. While our rules bypass the dynamics, they are derived from a consideration of the dynamics of extended fractures, with special attention being paid to the final overshoot state of stress. In our quasidynamic model we simulate the overshoot property in as general a way as possible without actually solving

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the dynamics of the complex system; the model can be iterated easily. By comparing the dynamic crack model with the quasidynamic model derived from it, we adduce the importance of dynamic overshoot in regulating earthquake self-organization. Since the dynamic crack models are much more computationally intensive than their quasistatic counterparts, our quasistatic model with appropriate overshoot will provide computational relief while preserving the requisite element of the dynamics.

Our vehicle for constructing a more appropriate quasistatic model is the one-dimensional dynamic Burridge-Knopoff (BK) [3] model, in which an earthquake is considered to be a dynamically evolving, extended fracture in fast time. The dynamic slip u_n between the walls of a homogeneous one-dimensional earthquake fault at a discrete lattice site n is the solution to the coupled ODEs

$$m\ddot{u}_n+k(2u_n-u_{n-1}-u_{n+1})+lu_n+\alpha\dot{u}_n=f_n\ , \tag{1}$$

where the velocity of sound in the continuum is $(k/m)^{1/2}a$, and a is the lattice spacing. Knopoff, Landoni, and Abinante [5] have given an argument for choosing $\alpha=2(lm)^{1/2}$, and give details of an efficient and accurate method of calculation. We seek an approximate solution to the above dynamic equations for the slip U_n at the end of any fracture event. We approximate U_n by the solution to (1) in its static form by deleting the time-dependent terms

$$k(2U_n-U_{n-1}-U_{n+1})+lU_n=f_n(1+\phi_N)\ , \tag{2}$$

where ϕ_N is an overshoot factor that depends on the length of the crack N, assumed to be the same for all particles. If there is no frictional damping ($\alpha=0$), the overshoot factor is $\phi_N=1$ for all N. In the usual quasistatic approximation, the final force is taken to be the static force (stress) drop, and $\phi_N=0$ for all N.

We assume that at some instant in the dynamical phase of fracture growth the length of the crack is N, i.e., N lattice sites are slipping. We number the lattice sites in motion from one end, $n=1,\ 2,\ldots,N$. To find an approximate value for ϕ_N, we assume that the displacement during fracture is in the lowest mode of the solution to (1), an assumption that has been shown to be valid with high accuracy in our numerical solutions of dynamical lattice models for selected distributions of threshold fracture strength and static stress drops. In this case, the solution to (1) is

$$u_n\propto\sin(n\gamma_1 a)[1-\cos(\Omega t)]e^{-(\alpha t/2m)}\ , \tag{3}$$

where $\Omega=2(k/m)^{1/2}\sin(\gamma_1 a/2)$, and γ_p is the wave number in the pth mode,

$$\gamma_p=\frac{p\pi}{(N+1)a}\ .$$

Under the freezing condition $\dot{u}_n=0$, the duration of the rupture is

$$\frac{\pi/2}{\left[\frac{k}{m}\right]^{1/2}\sin(\gamma_1 a/2)}\ .$$

Thus the overshoot factor is

$$\phi_N=\exp\left[-\left[\frac{l}{k}\right]^{1/2}\frac{\pi/2}{\sin\frac{\pi}{2(N+1)}}\right]\ ; \tag{4}$$

the case $N>>1$ can be derived as an obvious limit.

The solution to (2) is

$$kU_p=M_{pq}f_q(1+\phi_N)\ , \tag{5}$$

where

$$M_{pq}=\frac{\cosh(N+1-|p-q|)\psi-\cosh(N+1-p-q)\psi}{2\sinh\psi\sinh(N+1)\psi}\ , \tag{6}$$

and where

$$\cosh\psi=1+\frac{l}{2k}\ .$$

We especially note the force at the end of the fractured chain,

$$kU_1=\frac{\sum_q f_q(1+\phi_N)\sinh(N+1-q)\psi}{\sinh(N+1)\psi}\ . \tag{7}$$

Without loss of generality, we can set the dynamic friction on each particle to zero. Our algorithm is as follows.

(1) Increasing the driving stress slowly at a constant rate by increasing the force T_n on all particles uniformly until the difference between the fracture strength B_n and the force is zero at some site; fracture is initiated at this site. We set $f_n=T_n$ at the time of fracture.

(2) The crack grows progressively starting from a crack of length 1. Assume the crack has grown to a length N. Number the broken elements from 1 to N starting from one end.

(3) Solve for the forces at the ends of the crack (kU_1,kU_N) from (7). If either or both of the inequalities

$$kU_1>(B_0-T_0)\ \text{ and }\ kU_N>(B_{N+1}-T_{N+1})$$

are satisfied, then the length of the crack is increased to $(N+1)$ or $(N+2)$. Return to step (2).

(4) If neither inequality in step (3) is satisfied, this fracture event is assumed to have terminated. Set the final state of stress to $-f_n\phi_N$ for fractured sites and $(T_0+kU_1,T_{N+1}+kU_N)$ for the unbroken sites at the ends. Set $T_n=f_n$ for all the other sites. Return to step (1).

In Figs. 1 and 2 we compare the results of dynamic simulations (1) on a BK model with the quasidynamic algorithm (5) for three values of ϕ_N in each case: the values are ϕ_N, given by (4); $\phi_N=1$, which is the max-

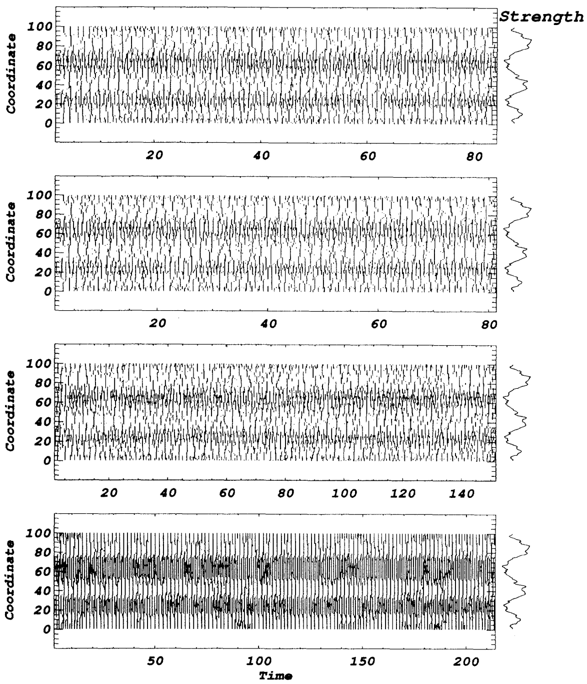

FIG. 1. Seismicity on a homogeneous one-dimensional BK fault model with periodic boundary conditions. The static friction is inhomogeneous, as shown at the right; the ratio of maximum to minimum friction is 5. Vertical strokes indicate the length of an individual fracture. The time scale is the same for all simulations: 3000 events are displayed in all cases (a) Dynamic case. (b) Quasidynamic case with overshoot given by Eq. (4). (c) Maximum overshoot $\phi_N = 1$. (d) Quasistatic case with zero overshoot.

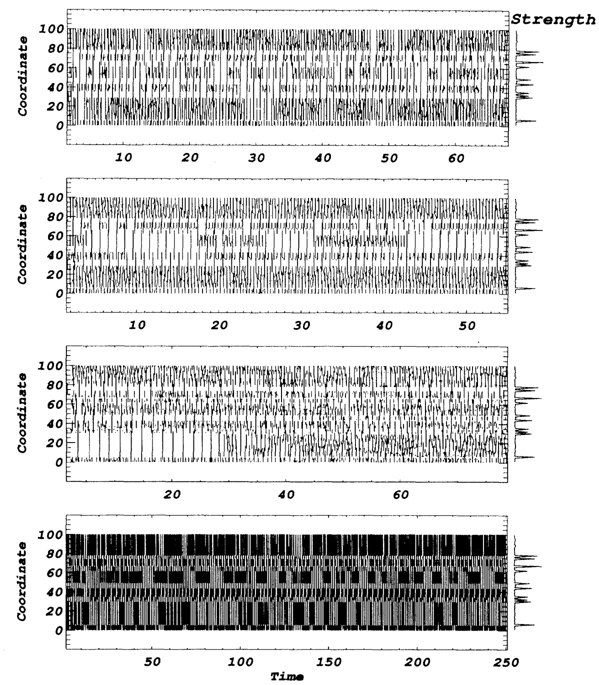

FIG. 2. Seismicity on a fault model as in Fig. 1, but with spiky inhomogeneous frictions as shown at right; the ratio of maximum to minimum frictions is 5 as before, and the frictions in the valleys vary between 1.0 and 1.2. The high density of events in (d) is due to the comparative absence of short fractures.

imum possible overshoot; and $\phi_N = 0$, which is the quasistatic approximation of no overshoot. The parameters for all four simulations in each figure are the same. The time scales are the same in each figure.

In Fig. 1 we display a simulation of 3000 events for the case $l/k = 0.25$, i.e., $\alpha = 1$, and $k = m = 1$. All lattice sites have the same properties, except for the distribution of fracture strengths, which is inhomogeneous as shown at the right; we use periodic boundary conditions. The problem is similar to that described in [5]. We display the space-time history of fractures; the vertical strokes in these displays represent the linear extent of the fractures without regard to the energy or moment released in the events. The localization in this model has been discussed in [5]. The reproduction of the dynamical result in Fig. 1(a) is of highest fidelity if the quasidynamic algorithm is used [see Fig. 1(b)]. The result with the quasistatic algorithm with no overshoot [Fig. 1(d)] is a poor reproduction of the dynamical result. Inspection of Fig. 1(c) shows that the case with too much overshoot $\phi_N = 1$ gives a better fit than the quasistatic case of zero overshoot.

Similar conclusions are reached in the second set of examples (Fig. 2), which is a study of a relatively smooth frictional system, punctuated by a series of spiky barriers of high fracture strength. Again we use periodic boundary conditions. The localization in this model has a significantly different character from that of the preceding figures; the dynamic solution [Fig. 2(a)] shows that the system tends to develop fractures that tear from barrier to barrier; these localized or characteristic events are frequently interrupted by stronger events that tear through the barriers. There are significant episodic shifts in the frequency of repetition of characteristic events locally. As before, the quasidynamic model with overshoot given by (5) gives the best reproduction [Fig. 2(b)], while the standard quasistatic model with zero-overshoot [Fig. 2(d)] gives the poorest results.

We find that the quasidynamic model with zero overshoot performs poorly in the simulation of iterative fractures in comparison with the dynamic model, whereas, with an appropriate choice of overshoot, the quasidynamic model reproduces with surprising qualitative accuracy the dynamic earthquake history. This is accomplished with an approximately $10N$-fold decrease in computing time over the dynamic equivalent model, where N is the lattice size of the largest ruptures that we model. These results lead us to conclude that overshoot due to dynamic healing is critically important in determining earthquake self-organization, and must be reckoned with in any effort to accurately model the physics of earthquake phenomena. Though the scaling of stress with distance from the fracture is not properly simulated in BK models with nearest-neighbor coupling, nevertheless the implications of the demonstration in this paper is that quasistatic models that take into account continuum elasticity will have to model overshoot appropriately as well.

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